

YANG-MILLS GRAVITY IN FLAT SPACE-TIME, II. GRAVITATIONAL RADIATIONS AND LEE-YANG FORCE FOR ACCELERATED COSMIC EXPANSION

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Within Yang-Mills gravity with translation group $T(4)$ in flat space-time, the invariant action involving quadratic translation gauge-curvature leads to quadrupole radiations which are shown to be consistent with experiments. The radiation power turns out to be the same as that in Einstein's gravity to the second-order approximation. We also discuss an interesting physical reason for the accelerated cosmic expansion based on the long-range Lee-Yang force of $U_b(1)$ gauge field associated with the established conservation law of baryon number. We show that the Lee-Yang force can be related to a linear potential $\propto r$, provided the gauge field satisfies a fourth-order differential equation. Furthermore, we consider an experimental test of the Lee-Yang force related to the accelerated cosmic expansion. The necessity of generalizing Lorentz transformations for accelerated frames of reference and accelerated Wu-Doppler effects are briefly discussed.

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1 Introduction

Right after the creation of the Yang-Mills theory in 1954, Utiyama immediately generalized the gauge field with $SU(2)$ group to a general symmetry

group with N generators and proposed the gauge-invariant interpretation of all interactions.[1, 2] In his pioneer work, Utiyama paved the way for far-reaching research on gauge theories of gravity and quantum gravity. In such gauge theories of gravity, Utiyama and others followed the usual approach of general relativity to formulate their theories within the framework of curved spacetime. However, the quantum aspect of general relativity in curved spacetime encountered long-standing difficulties because there is hardly any common ground between general relativity and quantum mechanics, as Wigner put it.[3] Dyson also stressed that the most glaring incompatibility of concepts in contemporary physics is that between the principle of general coordinate invariance and a quantum-mechanical description of all nature.[4] This incompatibility motivates the present investigation of Yang-Mills gravity with translation gauge symmetry within the framework of flat spacetime.

Yang-Mills gravity is logically independent of the conventional theory of gravity (or general relativity). The formulation of Yang-Mills gravity is based on bona fide local field theory in flat spacetime rather than in curved spacetime. Thus Einstein's field equation and Riemannian (or Robertson-Walker) metric are not postulate and, hence, cannot be used in the present formulation of gravity.

The theory follows a close analogy with the gauge invariant electrodynamics. Namely, the relevant symmetry leads, via Noether theorem, to a conserved quantity which is precisely the source for generating the field:

U(1) symmetry \rightarrow conserved electric charge \rightarrow the source of the electromagnetic potential field A_μ .

[with the replacement $\partial_\mu \rightarrow \partial_\mu - ieA_\mu$].

T(4) spacetime translation symmetry \rightarrow conserved energy-momentum tensor \rightarrow the source of the gravitational field $\phi_{\mu\nu}$.

[with the replacement $\partial_\mu \rightarrow \partial_\mu - ig\phi_\mu^\nu(i\partial_\nu)$, $i\partial_\nu =$ generators of T(4)].[5]

In a recent work (paper I),[5] I follow Yang-Mills' approach for internal gauge groups to discuss a generalized gauge theory for the 'external' spacetime translation group, whose generators do not have constant matrix representations, in sharp contrast to the internal groups. The above analogy between the internal group U(1) and the external spacetime group T(4) reveals two fundamental differences regarding electromagnetic and gravitational forces:

(i) Although the electromagnetic coupling constant e is dimensionless ($c = \hbar = 1$), the basic gravitational coupling constant g must have the dimension of length.

(ii) While the electromagnetic force involves both attractive and repulsive forces, the gravitational force can have only one of them.[5]

The present formulation of gauge theory with a translation gauge symmetry differs from all previous formulations with translation symmetry[6, 7] because (a) it is formulated within the framework of flat spacetime, and (b) it involves the Yang-Mills-type Lagrangian with quadratic gauge curvature, so that the interaction vertices of gravitons in Feynman diagrams are much simpler than those in general relativity. Thus, the present theory of Yang-Mills gravity has a common ground with quantum mechanics and quantum theory of gauge fields, and can be quantized just as usual fields. It is expected that the $T(4)$ gauge symmetry in flat spacetime will help to reduce the degrees of divergence in higher-order Feynman diagrams of Yang-Mills gravity and, hence, has a better high-energy behavior than that of general relativity. I have shown that the gauge-invariant action with quadratic gauge-curvature in flat space-time can produce good agreement with classical experiments such as the perihelion shift of Mercury, bending of light and so on.[5] In this paper, I present the calculation of gravitational quadrupole radiation and show its consistency with observations in section 2. Furthermore, to be consistent with the recent discovery of the accelerated cosmic expansion, I also consider the baryonic $U_b(1)$ gauge field which can produce a repulsive long-range force between baryon matter in flat spacetime. The gauge group for the whole theory is $T(4) \times U_b(1)$. Such a long-range ‘Lee-Yang force’ can be the physical reason for the accelerated cosmic expansion, because there is a modified $U_b(1)$ gauge invariant Lagrangian which leads to a linear cosmic potential $\propto r$. This is discussed in section 4. In section 5, we discuss an experimental test of the cosmic Lee-Yang force based on the accelerated Wu-Doppler effect.

2 Gravitational Quadrupole Radiations

Yang-Mills gravity is formulated for general frames of reference (i.e., inertial and non-inertial frames).[5] For simplicity, we choose inertial frames (in which $P_{\mu\nu} = \eta_{\mu\nu} = (+, -, -, -)$ and $D_\mu = \partial_\mu$) to discuss the gravitational quadrupole radiation. Ordinary matter contains baryons such as protons and neutrons (or up- and down-quarks). Let us consider the $T(4) \times U_b(1)$ gauge invariant action $S = \int L d^4x$. The Lagrangian L involves the gravitational tensor field $\phi_{\mu\nu}$, a (baryonic) fermion field ψ and the $U_b(1)$ gauge field associated with the conserved baryon numbers, [5]

$$L = \frac{1}{2g^2} \left(C_{\mu\alpha\beta} C^{\mu\beta\alpha} - C_{\mu\alpha}{}^\alpha C^{\mu\beta}{}_\beta \right)$$

$$+\frac{i}{2} [\bar{\psi}\gamma^\mu(\Delta_\mu\psi - ig_b B_\mu\psi) - (\Delta_\mu\bar{\psi} + ig_b B_\mu\bar{\psi})\gamma^\mu\psi] - m\bar{\psi}\psi + L_B, \quad (1)$$

$$L_B = -\frac{L_s^2}{4}\Delta_\lambda B_{\mu\nu}\Delta^\lambda B^{\mu\nu}, \quad B_{\mu\nu} = \Delta_\mu B_\nu - \Delta_\nu B_\mu, \quad (2)$$

$$C^{\mu\nu\alpha} = J^{\mu\lambda}(\partial_\lambda J^{\nu\alpha}) - J^{\nu\lambda}(\partial_\lambda J^{\mu\alpha}), \quad C_{\mu\alpha\beta}C^{\mu\beta\alpha} = (1/2)C_{\mu\alpha\beta}C^{\mu\alpha\beta},$$

$$\Delta_\mu\psi = J_{\mu\nu}\partial^\nu\psi, \quad J_{\mu\nu} = \eta_{\mu\nu} + g\phi_{\mu\nu} = J_{\nu\mu}, \quad c = \hbar = 1.$$

For the gravitational quadrupole radiation, we consider only the tensor field $\phi_{\mu\nu}$ and ignore the $U_b(1)$ gauge field B_μ because its coupling to baryon matter is much more weaker than that of gravitation. Moreover, it suffices to calculate the gravitation radiations to the second order in $g\phi_{\mu\nu}$. As usual, we impose the gauge condition

$$\partial^\mu\phi_{\mu\nu} = \frac{1}{2}\partial_\nu\phi^\lambda_\lambda. \quad (3)$$

The gauge invariant action with the Lagrangian (1) leads to the gravitational tensor field equation in inertial frames,[5]

$$H^{\mu\nu} = g^2 T^{\mu\nu}, \quad (4)$$

$$H^{\mu\nu} \equiv \partial_\lambda(J^\lambda_\rho C^{\rho\mu\nu} - J^\lambda_\alpha C^{\alpha\beta}_\beta \eta^{\mu\nu} + C^{\mu\beta}_\beta J^{\nu\lambda}) \\ - C^{\mu\alpha\beta}\partial^\nu J_{\alpha\beta} + C^{\mu\beta}_\beta \partial^\nu J^\alpha_\alpha - C^{\lambda\beta}_\beta \partial^\nu J^\mu_\lambda,$$

where μ and ν should be made symmetric and we have used the identities

$$C_{\mu\alpha\beta} = -C_{\alpha\mu\beta}, \quad C_{\mu\alpha\beta} + C_{\alpha\beta\mu} + C_{\beta\mu\alpha} = 0. \quad (5)$$

It is not necessary to write this symmetry of μ and ν in (4) explicitly for the following discussions of gravitational radiations. The energy-momentum tensor $T^{\mu\nu}$ in equation (4) is given by

$$T^{\mu\nu} = \frac{1}{2} [\bar{\psi}i\gamma^\mu\partial^\nu\psi - i(\partial^\nu\bar{\psi})\gamma^\mu\psi]. \quad (6)$$

For weak fields in inertial frames with the gauge condition (3), the field equation can be linearized as follows:

$$\partial_\lambda\partial^\lambda\phi^{\mu\nu} - \partial^\mu\partial_\lambda\phi^{\lambda\nu} + \partial^\mu\partial^\nu\phi^\lambda_\lambda - \partial^\nu\partial_\lambda\phi^{\lambda\mu} = g(T^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}T^\lambda_\lambda), \quad (7)$$

where we have used $J^{\mu\nu} = \eta^{\mu\nu} + g\phi^{\mu\nu}$. With the help of the gauge condition (3), (7) can be written as

$$\partial_\lambda\partial^\lambda\phi_{\mu\nu} = g(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^\lambda_\lambda) \equiv gS_{\mu\nu}, \quad g = \sqrt{8\pi G_N}, \quad (8)$$

where G_N is the Newtonian gravitational constant. To the first-order approximation, we obtain[5]

$$g\phi_{00} = g\phi_{11} = \frac{G_N m}{r}, \quad etc. \quad (9)$$

As usual, the energy-momentum tensor $T^{\mu\nu}$ is independent of $\phi^{\mu\nu}$ and satisfies the conservation law,

$$\partial_\mu T^{\mu\nu} = 0, \quad (10)$$

in the weak field approximation.

From equation (8), one has the usual retarded potential

$$\phi_{\mu\nu}(\mathbf{x}, t) = \frac{g}{4\pi} \int d^3x' \frac{S_{\mu\nu}(\mathbf{x}', t - |\mathbf{x} - \mathbf{x}'|)}{|\mathbf{x} - \mathbf{x}'|}, \quad (11)$$

$$\mathbf{x} \equiv \mathbf{r}, \quad \mathbf{x}' \equiv \mathbf{r}', \quad x^\mu = (w, x, y, z), \quad w = ct = t,$$

which is generated by the source $S_{\mu\nu}$ in (8). This equation is usually used to discuss the gravitational radiation. When one discusses the radiation in the wave zone at a distance much larger than the dimension of the source, the solution can be approximated by a plane wave,[8]

$$\phi_{\mu\nu}(x) = e_{\mu\nu} \exp(-ik_\lambda x^\lambda) + e_{\mu\nu}^* \exp(ik_\lambda x^\lambda), \quad (12)$$

where $e_{\mu\nu}$ is the polarization tensor. The plane wave property and the usual gauge condition (3) lead to

$$k_\mu k^\mu = 0, \quad k_\mu e_\nu^\mu = \frac{1}{2} k_\nu e_\mu^\mu, \quad k^\mu = \eta^{\mu\nu} k_\nu. \quad (13)$$

For the symmetric polarization tensor, $e_{\mu\nu} = e_{\nu\mu}$, of the massless tensor field in flat spacetime, there are only two physical states with helicity ± 2 which are invariant under the Lorentz transformation.

Let us write $T_{\mu\nu}(\mathbf{x}, t)$ in terms of a Fourier integral, [8]

$$T_{\mu\nu}(\mathbf{x}, t) = \int_0^\infty T_{\mu\nu}(\mathbf{x}, \omega) e^{-i\omega t} d\omega + c.c. \quad (14)$$

The retarded field emitted by a single Fourier component $T_{\mu\nu}(\mathbf{x}, t) = [T_{\mu\nu}(\mathbf{x}, \omega) e^{-i\omega t} + c.c.]$ is given by

$$\phi_{\mu\nu}(\mathbf{x}, t) = \frac{g}{4\pi} \int \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} S_{\mu\nu}(\mathbf{x}', \omega) \exp(-i\omega t + i\omega|\mathbf{x} - \mathbf{x}'|) + c.c. \quad (15)$$

$$S_{\mu\nu}(\mathbf{x}, \omega) = T_{\mu\nu}(\mathbf{x}, \omega) - \frac{1}{2} \eta_{\mu\nu} T(\mathbf{x}, \omega), \quad T = T^\lambda{}_\lambda.$$

The energy-momentum tensor $t_{\mu\nu}$ of gravitation is defined by the exact field equation (4) written in the following form,

$$\partial^\lambda \partial_\lambda \phi^{\mu\nu} = g(T^{\mu\nu} - t^{\mu\nu}). \quad (16)$$

Thus we have

$$\begin{aligned} t^{\mu\nu} = & \frac{1}{g^2} [C^{\rho\mu\nu} \partial_\lambda J_\rho^\lambda + \partial_\rho (g\phi^{\rho\lambda} \partial_\lambda J^{\mu\nu}) + g\phi_\rho^\lambda \partial_\lambda (J^{\rho\alpha} \partial_\alpha J^{\mu\nu}) \\ & - J_\rho^\lambda \partial_\lambda (J^{\mu\alpha} \partial_\alpha J^{\rho\nu}) - C^{\mu\beta\alpha} \partial^\nu J_{\alpha\beta} - \eta^{\mu\nu} \partial_\lambda (J_\rho^\lambda C^{\rho\beta}_\beta) + \partial_\lambda (C^{\mu\beta}_\beta J^{\nu\lambda}) \\ & + C^{\mu\beta}_\beta \partial^\nu J^\lambda_\lambda - C^{\lambda\beta}_\beta \partial^\nu J^\mu_\lambda]. \end{aligned} \quad (17)$$

To a second order approximation, we obtain the energy-momentum tensor of the gravitational field,

$$t^{\mu\nu} = t_1^{\mu\nu} + t_2^{\mu\nu}, \quad (18)$$

$$t_1^{\mu\nu} = (\partial_\lambda \phi) \partial^\lambda \phi^{\mu\nu} - \frac{1}{2} (\partial_\lambda \phi) \partial^\mu \phi^{\lambda\nu} + 2\phi^{\lambda\sigma} \partial_\lambda \partial_\sigma \phi^{\mu\nu} - (\partial_\lambda \phi^{\mu\sigma}) \partial_\sigma \phi^{\lambda\nu} \quad (19)$$

$$- \phi_\rho^\lambda \partial_\lambda \partial^\mu \phi^{\rho\nu} - \frac{1}{2} \phi^{\mu\sigma} \partial_\sigma \partial^\nu \phi - (\partial^\mu \phi^{\beta\alpha}) \partial^\nu \phi_{\alpha\beta} + (\partial^\beta \phi^{\mu\alpha}) \partial^\nu \phi_{\alpha\beta},$$

$$t_2^{\mu\nu} = -\frac{3}{4} (\partial_\lambda \phi) \partial^\lambda \phi \eta^{\mu\nu} - \phi^{\lambda\sigma} \partial_\lambda \partial_\sigma \phi \eta^{\mu\nu} + (\partial_\lambda \phi^{\beta\sigma}) \partial_\sigma \phi^\lambda_\beta \eta^{\mu\nu} \quad (20)$$

$$+ \frac{1}{2} (\partial^\nu \phi^{\mu\lambda}) \partial_\lambda \phi + \frac{1}{2} \phi^{\nu\lambda} \partial^\mu \partial_\lambda \phi + \phi^{\mu\sigma} \partial_\sigma \partial^\nu \phi$$

$$- (\partial^\nu \phi^{\beta\sigma}) \partial_\sigma \phi^\mu_\beta - \phi^{\beta\sigma} \partial_\sigma \partial^\nu \phi^\mu_\beta + \frac{3}{4} (\partial^\nu \phi) \partial^\mu \phi,$$

where $\phi \equiv \phi^\lambda_\lambda$, and we have used the gauge condition (3). The energy-momentum tensor $t_1^{\mu\nu}$ and $t_2^{\mu\nu}$ are respectively contributed from the first and the second quadratic gauge-curvatures (i.e., $C_{\mu\alpha\beta} C^{\mu\beta\alpha}$ and $-C_{\mu\alpha}{}^\alpha C^{\mu\beta}_\beta$ respectively) in the Lagrangian (1). One can use the plane wave solution (12) and the gauge condition (13) to calculate the energy-momentum tensors (18) in an inertial frame. This complicated result can be simplified by taking the average of $t^{\mu\nu}$ over a region of space and time much larger than the wavelengths of the radiated waves.[8] After such an average, one obtains the following results:

$$\langle t_1^{\mu\nu} \rangle = -2k^\mu k^\nu e^{\lambda\rho} e_{\lambda\rho}^* + \frac{1}{2} k^\mu k^\nu e_\lambda^\lambda e_\alpha^{*\alpha}. \quad (21)$$

$$\langle t_2^{\mu\nu} \rangle = \frac{1}{2} k^\mu k^\nu e_\lambda^\lambda e_\alpha^{*\alpha}.$$

Suppose one observes this radiation in the wave zone, one can write the polarization tensor in terms of the Fourier transform of $T_{\mu\nu}$:

$$e_{\mu\nu}(\mathbf{x}, \omega) = \frac{g}{4\pi r} [T_{\mu\nu}(\mathbf{k}, \omega) - \frac{1}{2}\eta_{\mu\nu}T(\mathbf{k}, \omega)], \quad T = T_\lambda^\lambda, \quad (22)$$

$$T_{\mu\nu}(\mathbf{k}, \omega) \equiv \int d^3\mathbf{x}' T_{\mu\nu}(\mathbf{x}', \omega) \exp(-i\mathbf{k} \cdot \mathbf{x}'), \quad (23)$$

$$\phi_{\mu\nu}(\mathbf{x}, t) \approx e_{\mu\nu}(\mathbf{x}, \omega) \exp(-ik_\lambda x^\lambda) + c.c., \quad (24)$$

where we have used (13) and (14) with the approximation $|\mathbf{x} - \mathbf{x}'| \approx r - \mathbf{x}' \cdot \mathbf{x}/|\mathbf{x}|$ and $\mathbf{k} = \omega\mathbf{x}/|\mathbf{x}|$ in the wave zone. Thus, the average energy-momentum of a gravitational plane wave can be written as

$$\langle t^{\mu\nu} \rangle = -\frac{G_N}{\pi r^2} k^\mu k^\nu \left(T^{\lambda\rho}(\mathbf{k}, \omega) T_{\lambda\rho}^*(\mathbf{k}, \omega) - \frac{1}{2} T(\mathbf{k}, \omega) T^*(\mathbf{k}, \omega) \right). \quad (25)$$

The power P_o emitted per unit solid angle in the direction $\mathbf{x}/|\mathbf{x}|$ is [8]

$$\frac{dP_o}{d\Omega} = r^2 \frac{x^i \langle t_{i0} \rangle}{|\mathbf{x}|}. \quad (26)$$

It can be written in terms of $T(\mathbf{k}, \omega)$ in (23),

$$\frac{dP_o}{d\Omega} = \frac{G_N \omega^2}{\pi} \left(T^{\lambda\rho}(\mathbf{k}, \omega) T_{\lambda\rho}^*(\mathbf{k}, \omega) - \frac{1}{2} T(\mathbf{k}, \omega) T^*(\mathbf{k}, \omega) \right). \quad (27)$$

Although the energy-momentum tensor of the gravitational field (18) in Yang-Mills gravity is quite different from that in general relativity, the result (27) for the power emitted per solid angle turns out to be the same as that obtained in general relativity and consistent with experiments.[8, 9] Following the usual method and approximation, one can calculate the power radiated by a body rotating around one of the principal axes of the ellipsoid of inertia. At twice the rotating frequency Ω , i.e., $\omega = 2\Omega$, one obtains the total power $P_o(\omega)$ emitted by a rotating body:

$$P_o(2\Omega) = \left[\frac{32G_N \Omega^6 I^2 e_q^2}{5} \right], \quad c = 1. \quad (28)$$

where I and e_q are respectively moment of inertia and equatorial ellipticity. Thus, to the second order approximation, the gravitational quadrupole radiation (28) predicted by the Yang-Mills gravity is also the same as that predicted by general relativity.[8]

3 Accelerated Cosmic Expansion and the Effective Repulsive Force due to Cosmological Constant

The discovery of the accelerated cosmic expansion stimulated many discussions.[10] The physical origin of a new ‘repulsive force’ (in the Newtonian approximation) has not been established. We would like to discuss and compare two of suggestions for the physical origin of the new repulsive force: (A) the cosmological constant in Einstein’s field equation, and (B) the Lee-Yang gauge field associated with the conserved baryon number. In this section, we first review and discuss the effective repulsive force due to the cosmological constant from the viewpoint of field theory.

It is reasonable to expect that in a non-relativistic approximation of a theory, apart from the usual Newtonian gravitational force, there is an additional repulsive force between two ordinary objects. The total ‘cosmic force’ F_C between two objects can be written phenomenologically as a combination of the usual gravitational attractive force and another long-range repulsive force $Bf(r)$,

$$F_C \approx -\frac{GM_1M_2}{r^2} + Bf(r), \quad (29)$$

where B denotes the strength of the new long-range force.

Experimentally, this new force in (29) must be very much smaller than the Newtonian gravitational force in the solar system and in our galaxy because it has not been detected. It appears that this new force becomes important only in a very large cosmic scale.

Although we do not assume general relativity or Einstein’s equation in the formulation of Yang-Mills gravity, it is interesting to compare it and the corresponding results in the present theory. Einstein’s field equation with the cosmological constant is given by

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - \lambda g^{\mu\nu} = -8\pi GT_E^{\mu\nu}. \quad (30)$$

In order to get a simple picture for the role played by the cosmological constant λ , let us consider the static Newtonian approximation of (30):[11]

$$\nabla^2\phi = m\delta^3(\mathbf{r}) + \lambda, \quad (31)$$

for a mass point located at the origin. We have used $T_E^{00} = m\delta^3(\mathbf{r})$ and $g^{00} = 1 - 2\phi$. The spherically symmetric solution to (31) is given by

$$\phi = \phi_g + \phi_c, \quad \phi_g = -\frac{Gm}{r}, \quad \phi_c = \frac{\lambda}{6}r^2, \quad (32)$$

where $\lambda < 0$ corresponds to a cosmic repulsive force in a classical limit. The cosmological constant λ in (31) behaves like an undetectable ‘new ether’ with a constant density everywhere in the universe. It acts like a strange source which generates a new cosmological potential $\phi_c = \lambda r^2/6$.

As a result, the equation of motion of a freely moving test particle in the Newtonian limit is changed to the following form

$$\frac{d^2 \mathbf{r}}{dt^2} = \mathbf{g} - C \mathbf{r}, \quad (33)$$

where \mathbf{g} is the gravitational acceleration produced by the distribution of ordinary matter, while the ‘dark energy’ acceleration $\propto \mathbf{r}$ is due to a constant ‘dark energy’ density $\Omega_{\Lambda o}$ everywhere in the universe ($C = \Omega_{\Lambda o} H_o^2$ and H_o =Hubble constant).[10]

Einstein originally believed (in 1917) that the large-scale structure of the universe is static, so he introduced the term with the cosmological constant in his field equation (30) to be consistent with his belief. However, this static solution for a universe is unstable. This property can be seen in equation (33), the mass distribution can be chosen such that the two terms cancel so that $d^2 \mathbf{r}/dt^2 = 0$. However, the cancellation of these two terms in (33) can be easily upset by a redistribution of masses in the universe or by a small perturbation to the mean mass density.[10] Eventually, Einstein gave up the cosmological constant for two reasons:(i) logical economy, and (ii) Hubble’s discovery of the expansion of the universe.

Nevertheless, the presence of the cosmological constant λ in Einstein’s field equation (30) is now postulated by many people to be the cause of the observed accelerated expansion of the universe. Although Einstein did not consider the cosmological constant to be part of the energy-momentum tensor, it is equivalent to consider it as part of the energy-momentum tensor, i.e., a ‘new component’ in the content of the universe.[10] But from the viewpoint of field theory, the presence of the ‘additional’ source term λ in (31) turns out to be very strange. Equation (31) suggests that the potential field ϕ (or $g_{00} = 1 + 2\phi$) is generated by two distinct sources, $m\delta^3(\mathbf{r})$ and λ , at the same time. Nevertheless, so far all known experiments show that different kinds of sources generate different kinds of fields in field theory and particle physics. Furthermore, this type of non-local source will probably cause further difficulty in the quantization of field in (30). For example, the inverse-square force corresponding to the potential Gm/r has the field-theoretic interpretation, namely, it is due to the exchange of virtual gravitons described by the field equation (30), which can be seen in Feynman’s discussion.[12] On the contrary, the linear force in (33) corresponds to the cosmological potential $r^2\lambda/6$ and does not have a field-theoretic interpretation based on the field equation (30).

4 Accelerated Cosmic Expansion and the Lee-Yang Force Associated with Conserved Baryonic Charge

The theory of Yang-Mills gravity is based on the translation gauge group $T(4)$ in flat spacetime and does not allow a cosmological constant. The motivation to find a more natural explanation for accelerated cosmic expansion led us to investigate the cosmic Lee-Yang repulsive force associated with the observed conservation of baryon numbers (or baryon charges) through the principle of gauge symmetry.[13] For this purpose, the Yang-Mills gravity is extended to include the $U_b(1)$ gauge field, so that the whole theory is based on the gauge group $T(4) \times U_b(1)$.

Soon after the creation of Yang-Mills theory of $SU(2)$ gauge field related to isospin conservation, Lee and Yang discussed in 1955 a long-range repulsive force ($\propto 1/r^2$) between baryons based on the $U_b(1)$ gauge symmetry associate with the experimentally established conservation of baryon charge (or number).[14] Using Eötvös experiment, the strength of such a repulsive force between nucleons (or baryons) was estimated to be at least one million times smaller than that of the gravitational force. Such an extremely weak inverse-square force will probably never be observed. Nevertheless, we discuss a modified gauge invariant Lagrangian for the $U_b(1)$ gauge field, which suggests a new r -independent cosmological force $Bf(r)$ in (29) between observable galaxies (which are assumed to be made of baryons and leptons). We suggest that such a new r -independent cosmic force can be produced by the gauge fields associated with baryon numbers and electron-lepton numbers.[13] The conservation of these quantum numbers has been experimentally established.[15]

We observe that the gauge invariant Lagrangian for massless $U_b(1)$ gauge field is, strictly speaking, not unique. The reason is that besides the usual Lagrangian which is quadratic in the fields strength $B_{\mu\nu}$, there is another simple gauge invariant Lagrangian which is quadratic in $\partial_\lambda B_{\mu\nu}$. [13] This simple gauge invariant Lagrangian is interesting because it can lead to a ‘linear potential’, $\propto r$, which differs from the ‘quadratic potential’ in (32) associated with the cosmological constant.

Let us consider such a gauge invariant Lagrangian involving up and down quarks and baryonic gauge field B_μ for accelerated cosmic expansion (ACE):

$$L_{ACE} = -\frac{L_s^2}{4} \partial_\lambda B_{\mu\nu} \partial^\lambda B^{\mu\nu} + L_{ud}, \quad (34)$$

$$L_{ud} = i\bar{u}_n \gamma_\mu (\partial^\mu - \frac{ig_b}{3} B_\mu) u_n - m_u \bar{u}_n u_n + i\bar{d}_n \gamma_\mu (\partial^\mu - \frac{ig_b}{3} B_\mu) d - m_d \bar{d}_n d_n,$$

where the color index n is summed from 1 to 3. This is the part of the Lagrangian (1) with $U_b(1)$ gauge symmetry. As usual, it is not necessary to symmetrize the fermion Lagrangian with the $U_b(1)$ gauge symmetry.

One can easily include the gauge field associated with the conserved lepton numbers in (34). For simplicity, we shall not discuss it here. The new gauge-invariant field equation derived from (34) is a fourth-order partial differential equation of B_μ ,

$$\partial^2 \partial_\nu B^{\nu\mu} - g'_b J_q^\mu = 0, \quad g'_b = g_b/(3L_s^2), \quad (35)$$

where the source of the gauge field is

$$J_q^\mu = \bar{u}_n \gamma^\mu u_n + \bar{d}_n \gamma^\mu d_n.$$

Clearly, the field strength $B_{\mu\nu}$ satisfies the Bianchi identity

$$\partial^\lambda B^{\mu\nu} + \partial^\mu B^{\nu\lambda} + \partial^\nu B^{\lambda\mu} = 0. \quad (36)$$

For the static case, the field B_0 satisfies the fourth order differential equation

$$\nabla^2 \nabla^2 B^0 = -\frac{g_b}{3L_s^2} J^0 \equiv \rho_B. \quad (37)$$

The general solution to B^0 can be expressed in terms of a modified version of the usual Green's function for a second order partial differential equation,

$$B^0(\mathbf{r}) = \int F(\mathbf{r}, \mathbf{r}') \rho_B(\mathbf{r}') d^3 r', \quad (38)$$

$$F(\mathbf{r}, \mathbf{r}') = \frac{1}{\nabla^2} \frac{-1}{4\pi|\mathbf{r} - \mathbf{r}'|} = \int \frac{e^{-i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')}}{\mathbf{k}^2} \frac{d^3 k}{(2\pi)^3} \frac{1}{4\pi|\mathbf{r}'' - \mathbf{r}'|} d^3 r'',$$

where $1/\nabla^2$ is an integral operator.

Suppose we impose a 'Coulomb-like gauge' $\partial_k B^k = 0$, the static exterior potential satisfies the equation $\nabla^2 \nabla^2 B^0 = 0$. The solution for such a potential can be written in the form $B^0 = A'/r + B'r + C'r^2$. Different r -dependent terms in B^0 corresponds to sources with different types of singularity at $r = 0$ or different boundary conditions at infinity. For large distances, the last term $C'r^2$ dominates. In this case, the classical potential B_0 in (37) leads to the same effect as that of Einstein's cosmological constant and implies the modified gravitational law of motion (33). However, the potential $C'r^2$ is incompatible with the field equation (35) [with the usual current-source J_q^μ] from the viewpoint of field theory: There is simply no way to produce such a force through the exchange of virtual quantum between two Dirac's fermions carrying baryon charges. It turns out that

only the linear potential $B'r$ is due to the exchange of virtual quantum described by the fourth order field equation (37). Why? The reason is as follows:

From the viewpoint of quantum field theory, the fermion (baryon) source ρ_B in (37) is represented by the usual delta-function because these fermions satisfy the Dirac equations. Based on this property, we can show that the solution for the potential B_0 should be proportional to r rather than r^2 . This result can be seen explicitly by substituting $\rho_B(\mathbf{r}) = g'_b \delta^3(\mathbf{r})$ in (38), where $g' = g_b/(3L_s^2)$ for quarks and $g' = g_b/(L_s^2)$ for protons and neutrons. We obtain

$$B^0(\mathbf{r}) = g'_b \int_{-\infty}^{\infty} \frac{1}{(\mathbf{k}^2)^2} e^{i\mathbf{k}\cdot\mathbf{r}} d^3k = -\frac{g'_b}{8\pi} |\mathbf{r}|, \quad (39)$$

which can be understood as a generalized function.[16] It can also be obtained by using the relation

$$\int_0^{\infty} \frac{\sin ax}{x(b^2 - x^2)} dx = \frac{\pi}{2b^2} (1 - \cos ab), \quad (40)$$

and take the limit $b \rightarrow 0$. The limit appears to be not trivial because if the denominator $x(b^2 - x^2)$ in the integrand of (40) is replaced by $(b^4 + x^4)/x$, the integral exists, but the limit $b \rightarrow 0$ does not. Thus, it is more rigorous to treat the Fourier transform of linear potential and its inverse transform within the framework of generalized functions.[16]

The result (39) implies that the force associated with the linear potential $B^0(\mathbf{r})$ can be interpreted as the exchange of virtual quantum between two quarks, where the quantum satisfies the fourth-order field equation (37). The situation is the same as that in quantum electrodynamics. Namely, the static Coulomb potential produced by a point charge is the Fourier transform of the Feynman propagator of a virtual photon with $k_0 = 0$.

5 Experiments of Lee-Yang Force with the Wu-Doppler Effect

Let us consider a possible experimental tests of Lee-Yang force by measuring the accelerations of supernovae based on a modified Doppler Effect. First we note that, in analogy with (33), the constant Lee-Yang force will modify Newtonian law of motion for a test particle as follows:

$$\frac{d^2\mathbf{r}}{dt^2} = \mathbf{g} + \mathbf{g}_u, \quad (41)$$

It is convenient to use a nucleon rather than a quark with a baryonic charge $g_b/3$ for discussions. A nucleon carries a baryon charge g_b and a mass

m. Thus, the second term for two nucleons will be $|\mathbf{g}_u| = g_b/(8\pi L_s^2 m)$, which is the r -independent acceleration associated the cosmic linear potential $g_b^2 \mathbf{r}/(8\pi L_s^2)$ of the fourth-order field equation (37) [with $g_b/3$ replaced by g_b of a baryon]. It implies the existence of a constant acceleration \mathbf{g}_u between any two galaxies made of baryons, provided other forces (e.g., the ‘magnetic-type’ force) are negligible. This is an interesting prediction of $U_b(1)$ gauge symmetry together with the fourth-order gauge-invariant field equation (37). It is important to test the prediction (41).

Physically, the accelerated cosmic expansion[17] implies that, strictly speaking, there is no inertial frame in the universe. Inertial frames are merely idealized reference frames for simplification of discussions of the physical laws and phenomena. At least, one should seriously consider physics in a more realistic reference frames with linear accelerations by investigating a generalization of Lorentz transformations for accelerated frames. We have considered one of simple generalizations, called Wu transformations, for reference frames with constant-linear-accelerations (CLA) in the previous paper.[5] In terms of the differentials $dx_I^\mu = (dw_I, dx_I, dy_I, dz_I)$ for an ‘Inertial Frame’ $F_I(x_I^\mu)$ and $dx^\mu = (dw, dx, dy, dz)$ for a CLA frame $F(x^\mu)$, a simple generalization of the Lorentz transformation takes the following form:

$$dw_I = \gamma(Wdw + \beta dx), \quad dx_I = \gamma(dx + \beta Wdw) \quad (42)$$

$$dy_I = dy, \quad dz_I = dz;$$

where $W = \gamma^2(\gamma_o^{-2} + \alpha_o x)$, $\beta = \alpha_o w + \beta_o$, $\gamma_o = 1/\sqrt{1 - \beta_o^2}$, $\gamma = 1/\sqrt{1 - \beta^2}$. They can be integrated to obtain the transformation for x_I^μ and x^μ . [5] The Wu transformation (42) implies

$$ds^2 = dw_I^2 - dx_I^2 - dy_I^2 - dz_I^2 = W^2 dw^2 - dx^2 - dy^2 - dz^2 = P_{\mu\nu} dx^\mu dx^\nu. \quad (43)$$

where $P_{\mu\nu} = (W^2, -1, -1, -1)$ is the metric tensor for the CLA frame F . This simple form of the metric tensor $P_{\mu\nu}$ suggests that if one defines a distorted ‘Wu differential’, (Wdw, dx, dy, dz) , for the CLA frame $F(x^\mu)$, then the transformation of the ‘Wu differential’ from such a CLA frame to an inertial frame F_I will be formally the same as the Lorentz transformation, except that the constant velocity is replaced by a time-dependent velocity $\beta = \alpha_o w + \beta_o$, as shown in (42). Nevertheless, the accelerated frame $F(w, x, y, z)$ is not equivalent to the inertial frame $F_I(w_I, x_I, y_I, z_I)$ because the Wu differentials are distorted only for CLA frames due to its acceleration, but not for inertial frames.

However, it appears reasonable to assume that all reference frames with the same constant-linear-acceleration (CLA) are ‘equivalent’ in the following sense: Suppose F and F' are two such CLA frames. An atom H_e

at rest in F has the same properties as another H_e atom at rest in F' . For example, if an atom H_e at rest in F (F') emits a light wave with the wavelength λ_o (λ'_o), as immediately measured in F (F'); then we have the equality relation, $\lambda_o = \lambda'_o$. It appears to be reasonable to employ such an ‘equivalence’ for two CLA frames as a guiding principle for our discussions below.

To measure directly the accelerations of distant supernovae Ia, the usual Doppler effect is no longer adequate. One must know how the usual relation of the Doppler effect is modified by the linear accelerations of the light source and observers. In this connection, the accelerated Wu transformation will be very useful. Suppose a light source is at rest in the accelerated frame F , and the observer is, for simplicity, at rest in an inertial frame F_I . The covariant wave 4-vector k_μ has the same transformation as the covariant coordinate 4-vector $dx_\nu = P_{\nu\mu}dx^\mu$. We note that the coordinate x^μ of a CLA frame is no longer a 4-vector. Thus the Wu transformation implies the following accelerated Wu-Doppler effect:

$$k_{I0} = \gamma(W^{-1}k_0 - \beta k_1), \quad k_{I1} = \gamma(k_1 - \beta W^{-1}k_0) \quad (44)$$

$$k_{I2} = k_2, \quad k_{I3} = k_3;$$

for the covariant wave 4-vector $k_\mu = (k_0, k_1, k_2, k_3)$, where

$$(W^{-1}k_0)^2 - \mathbf{k}^2 = k_{I0}^2 - \mathbf{k}_I^2. \quad (45)$$

In the limit of zero acceleration, $\alpha_o \rightarrow 0$, the Wu transformation reduces to the Lorentz transformation, and the Wu-Doppler effect (44) becomes the usual relativistic Doppler effect in special relativity.

Presumably, the Lee-Yang force and the constant acceleration g_u in (41) due to the baryonic charge are extremely small, so that their dynamical effects in particle physics cannot be detected in high energy laboratories. Moreover, only gigantic bodies like galaxies separated by a great distance can have enough repulsive Lee-Yang force to overcome the gravitational attractive force and, hence, to move with an acceleration which may be detected through the Wu-Doppler effect of the wavelength in the radiation of a supernova Ia.

Suppose the earth, the supernova ‘a’ and the supernova ‘b’ are respectively at rest in the constant-linear-acceleration (CLA) frames F , F' and F'' , which are moving with velocities $\beta = \beta_o + \alpha_o w$, $\beta' = \beta'_o + \alpha'_o w'$, and $\beta'' = \beta''_o + \alpha''_o w''$ along the +x axis. The Wu-Doppler effect (44) can be applied to these three CLA frames. Suppose an atom at rest in F (F' , F'') emits a lights (propagating along the x-axis) with the wavelength λ_o (λ'_{oa} , λ''_{ob}), as immediately measured in F (F' , F''). We have $\lambda_o = \lambda'_{oa} = \lambda''_{ob}$ if the frames F , F' and F'' have the same acceleration or have very small

accelerations. Suppose these two different lights emitted from supernovae have the wavelength λ_a and (λ_b) , as measured on the earth (i.e., F frame). We obtain

$$\frac{\lambda_a - \lambda_b}{\lambda_a \lambda_b} = \frac{1}{\lambda_o \gamma (1 - \beta)} [\gamma'' (1 - \beta'') - \gamma' (1 - \beta')] \quad (46)$$

from (44) and (45). In general, the values of accelerations of the supernovae are extremely small and difficult to measure because the initial conditions of their motions in (46) are not known.[18] However, we are interested in testing the predictions (33) and (41), namely, whether the accelerations α'_o and α''_o are the same or not. To see the difference of these two cases based on the Wu-Doppler effect (46), we assume that, for simplicity, the Earth frame can be approximated by an inertial frame, $F = F_I$, and that the initial velocities of the frames F' and F'' are zero. Using the inverse Wu transformations, the velocities β' and β'' can be expressed in terms of quantities measured in the frame $F = F_I$, e.g., $\beta' = (\alpha_{oa} w_{Io} + \beta_o / \gamma_{oa}) / (\alpha_{oa} x_{Ia} + 1 / \gamma_{oa})$. [5, 19] For small velocities and accelerations, (46) can be approximated by

$$\frac{\lambda_a - \lambda_b}{\lambda_a \lambda_b} = \frac{1}{\lambda_o} \left[\frac{\alpha_{oa} w_{Io}}{1 + \alpha_{oa} x_{Ia}} - \frac{\alpha_{ob} w_{Io}}{1 + \alpha_{ob} x_{Ib}} \right], \quad (47)$$

where w_{Io} is the time of observation, and x_{Ia} and x_{Ib} are the distances of the supernovae ‘a’ and ‘b’ as measured in the Earth frame. If supernovae ‘a’ and ‘b’ have the same acceleration, $\alpha_{oa} = \alpha_{ob} = g_u$, the value of cosmic acceleration can be estimated through the measurement of λ_a and λ_b on the Earth, provided (A) the Earth can be considered as the F_I frame after some necessary and careful corrections and (B) w_{Io} is measured in cosmic time which is presumably the same order of magnitude as the age of the universe.

When one considers the motion of galaxies, the expansion and the accelerated expansion of the universe as a whole, one uses a cosmic time. Such a cosmic time appears to be quite different from the relativistic time of special relativity. However, within the four-dimensional symmetry framework, one can define a time for all observers in different frames to record time and to describe physics, and still preserve the Lorentz and Poincaré invariance of physics laws. Such a time is called ‘common time’ which resembles the cosmic time.[19, 20]

On the other hand, if the accelerations α_o , α'_o , and α''_o are not the same, it will be more difficult to determine the values of accelerations by the Wu-Doppler effect. Nevertheless, since all these accelerations are very small, (47) may be approximated by

$$\frac{\lambda_a - \lambda_b}{\lambda_a \lambda_b} = \frac{w_{Io}}{\lambda_o} [\alpha_{oa} - \alpha_{ob} + \alpha_{ob}^2 x_{Ib} - \alpha_{oa}^2 x_{Ia}]. \quad (48)$$

The qualitative difference between (33) and (41) can be tested by using (48), provided one has enough data and accuracy for the measurements of the wave lengths. Of course, before one applies the Wu-Doppler effect (44) to investigate the acceleration of cosmic expansion, one must also test (44) in the laboratory.

It is important to carry out many different kinds of experiments to test the difference between two interesting predictions (33) and (41) for the accelerated cosmic expansion. Qualitatively speaking, the universe was much smaller in an earlier era, so that the Lee-Yang repulsive force between two galaxies was overwhelmed by the usual attractive gravitational force. Thus we have decelerated cosmic expansion in an earlier era. As the intergalactic distances increase, the gravitational force decreases, and there will be a critical distance R_c where the Lee-Yang repulsive force cancels the gravitational attractive force between two galaxies. As an example, if one considers an isolated system of two baryons with baryon charge g_b and mass m , the critical distance R_c is given by

$$R_c = (Gm^2 8\pi L_s^2 / g_b^2)^{1/2}. \quad (49)$$

When the intergalactic distances became larger than the critical distance, the r -independent Lee-Yang repulsive force would overcome the gravitational attractive force, and one would have accelerated cosmic expansion. This appears to be what we have observed in the recent era of cosmic evolution.[17]

Comparing the difference of the two predictions (33) and (41), we could have the following scenario: As the intergalactic distance increases, the constant Lee-Yang force may be weaker than the linear force. One usually takes the data of distant supernovae Ia (with roughly the same intrinsic brightness) and plots relative apparent brightness (or relative brightness) against the redshift z to show the accelerated cosmic expansion at the present era. But the data also show that in an earlier epoch (in cosmic time) with a higher redshift z , one has deceleration.[17] In comparison with the linear force by using the graph of relative apparent brightness and redshift, the constant Lee-Yang force will probably give a smaller acceleration for the cosmic expansion for small redshift and give a larger deceleration for large redshift z (or earlier era). We stress that the experimental evidence for unambiguous r -dependence or r -independence of the cosmic acceleration is important because of the following reasons: If the prediction (33) of the cosmological constant is confirmed, then the current field theory and particle physics are inadequate for understanding physics at the cosmological scale. On the other hand, if the prediction (41) of the Lee-Yang force associated with conserved baryonic charge is confirmed, this result would imply that we do not have to make the unnatural assumption that about 70%

of the energy density of the universe is the unknown ‘Dark Energy’[17, 21] or a sort of ‘new ether’ everywhere in space. Furthermore, this would imply that we can understand cosmological phenomena based on field theory and particle physics, which are originally formulated for microscopic world. This would be a further support of the principle of gauge symmetry and an interesting unification of physics at microscopic and macro-cosmic worlds.

6 Discussions and Remarks

(A) Effective Metric Tensor

The basic Lagrangian for tensor fields and electromagnetic fields[5] within Yang-Mills gravity does not explicitly involve the effective Riemannian metric tensor. From the viewpoint of gauge symmetry, the Yang-Mills gravity in flat spacetime reveals the field-theoretic origin of an effective Riemannian metric tensor for the motion of classical particles. Namely, such an effective metric tensor shows up only in the limit of geometric optics (or classical limit) of wave equations for quantum particles. In other words, the Yang-Mills approach to gravity suggests that the underlying basis for gravity is the translation gauge symmetry in flat spacetime rather than the general coordinate invariance in curved spacetime. This property could shed light on quantum gravity, which will be discussed in a separate paper.

(B) Fundamental Length

The gauge group of the Yang-Mills gravity is $T(4) \times U_b(1)$, where $T(4)$ and $U_b(1)$ appear to be simply juxtaposed, in contrast to those in the unified electroweak theory. However, there are two basic constants with the dimension of length in this theory: One is the gravitational coupling constant g in the $T(4)$ -invariant Lagrangian, and the other is the length scale L_s in the $U_b(1)$ -invariant Lagrangian. It is quite possible that these two basic constants are not independent. And they could provide a profound relationship that reveals the fundamental length of nature.

(C) Linear Potentials for Accelerated Cosmic Expansion and for Quark Confinement

The physical significances of linear potential in field theory are [I] it can provide a constant repulsive force for the accelerated cosmic expansion (at extremely large distances), and [II] it can also provide an understanding of the permanent confinement of quarks and antiquarks inside hadrons (at very small distances), similar to Yukawa’s treatment for the short-range nuclear force. The property [II] is not trivial because if the gauge field is a vector field, then the linear potential will have both attractive and repulsive force. But the quark confinement requires the forces for quark-quark and quark-antiquark to be always attractive. One simple way to satisfy this

requirement appears to be that the field is, say, a scalar field[22] or a tensor field, similar to the spacetime translation gauge field.[5] It would be very interesting and significant if the potentials at the smallest distance scale between confined quarks [23] and at the largest scale between galaxies with accelerated expansion are both due to linear potentials associated with the fourth-order field equations. These properties deserve to be further studied.

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